Section 11-2

Using the Fundamental Counting Principle

Nov 6-7:09 AM

N = { Alan, Bill, Cathy, David, Evelyn }

1) How many ways can you select a president?

2) How many ways can you select a president and a secretary?

3) How many ways can you select a president, a secretary and a treasurer if the president must be a female and the other two must be male?

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Fundamental Counting Principal:

When a task consists of separate parts and satisfies the uniformity criterion, the total number of ways to complete the task is:

Formula: $n_1 \times n_2 \times \dots n_k$

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Example:

How many two digit natural numbers are there in our base ten system?

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Example:

Find the number of two digit numbers that do **not contain repeating digits**.

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Example:

How many two-digit numbers can be made from the set {0, 1, 2, 3, 4, 5}? (numbers can't start with 0.)

Solution

There are 5(6) = 30 two-digit numbers.

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Example:

How many two-digit numbers that **do not contain repeated digits(cannot start with a zero)** can be made from the set {0, 1, 2, 3, 4, 5} ?

Solution

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Creating an ID

How many ways can you create an ID with two letters followed by three digits?

Solution

There are 26(26)(10)(10)(10) = 676,000 IDs possible.

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Example:

How many non-repeating odd 3-digit natural (counting) numbers are there?

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In some states, license plates have 3 letters followed by 3 digits.

How many possible license plates are there?

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11.2 Continued

For any counting number n, the product of *all* counting numbers from n down through 1 is called ***n*** __________, and is denoted ***n*!**.

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Factorial Formula

For any counting number n, the quantity ***n*** **factorial** is given by

$$n! = n(n-1)(n-2)\ldots 2 \cdot 1.$$

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Examples:

Evaluate each expression.

a) $4!$ b) $(4-1)!$ c) $\frac{5!}{3!}$

Solution

a) $4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$

b) $(4-1)! = 3 \cdot 2 \cdot 1 = 6$

c) $\frac{5!}{3!} = \frac{5 \cdot 4 \cdot 3!}{3!} = 5 \cdot 4 = 20$

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Definition of Zero Factorial

$$0! = 1$$

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Warm -Up

1) In some states, license plates have 3 letters followed by 4 digits. How many possible license plates are there if letters cannot repeat and the first digit cannot be a zero?

2) Determine the number of outcomes for which the sum of rolling 2 dice is less than 5.

3) a) 7! b) (8-3)! c) 8!-3! d)

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Arrangements of Objects

When finding the total number of ways to *arrange* a given number of distinct objects, we can use a factorial.

The total number of different ways to arrange n distinct objects is $\boldsymbol{n!}$.

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Arranging Books

How many ways can you line up 6 different books on a shelf?

Solution

The number of ways to arrange 6 distinct objects is 6! = 720.

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The number of **distinguishable arrangements** of n objects, where one or more subsets consist of look-alikes is given by:

$$\frac{n!}{n_1!n_2!\cdots n_k!}.$$

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Example: Distinguishable Arrangements

Determine the number of distinguishable arrangements of the letters of the word INITIALLY.

Solution

9 letters total
3 I's and 2 L's

$$\frac{9!}{3!2!} = 30240.$$

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How many DISTINGUISHABLE arrangements can you create using the letters?

A) OCONNOREAGLE

B)CANDYCRUSH

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p. 690 7-25 odd, 27-31, 37-40, 57, 58 a, b

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Section 11-3

Permutations and Combinations

Nov 6-7:09 AM

Guidelines on Which Method to Use

Permutations	**Combinations**
Number of ways of selecting r items out of n items	
Repetitions are not allowed	
Order is important.	Order is not important.
Arrangements of n items taken r at a time	Subsets of n items taken r at a time
$nPr = \frac{n!}{(n-r)!}$	$nCr = \frac{n!}{(n-r)!r!}$
Clue words: arrangement, schedule, order, President, VP, 1st, 2nd	Clue words: group, subset, sample, selection, committee

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EXAMPLE - Identify as a Combination or a Permutation.

1. Telephone Number
2. Social Security Number
3. Poker Hand
4. A Committee of 5 chosen from a class of 10.
5. A combination lock
6. Powerball Numbers
7. License Plate

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Example: Poker

A common form of poker involves hands (sets) of five cards each, dealt from a deck consisting of 52 different cards. How many different 5-card hands are possible?

Solution

Repetitions are not allowed and order is not important.

$${}_{52}C_5 = \frac{52!}{5!(52-5)!} = \frac{52!}{5!47!} = 2,598,960$$

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Example: Number of Subsets

Find the number of different subsets of size 3 in the set $\{m, a, t, h, r, o, c, k, s\}$.

Solution

A subset of size 3 must have 3 distinct elements, so repetitions are not allowed. Order is not important.

$${}_9C_3 = \frac{9!}{3!(9-3)!} = \frac{9!}{3!6!} = 84$$

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Example: IDs

How many ways can you select two letters followed by three digits for an ID if repeats are not allowed?

Solution

There are two parts:

1. Determine the set of two letters.
2. Determine the set of three digits.

$$\underbrace{{}_{26}P_2}_{\text{Part 1}} \cdot \underbrace{{}_{10}P_3}_{\text{Part 2}} = 650 \cdot 720 = 468,000$$

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Example:

How many four-digit numbers can be written using the numbers from the set {1, 3, 5, 7, 9} if repetitions are not allowed?

Solution

Repetitions are not allowed and order is important, so we use permutations:

$$_5P_4 = \frac{5!}{(5-4)!} = \frac{5!}{1!} = 120.$$

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