Main Ideas

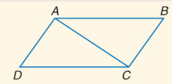
- Recognize and apply properties of the sides and angles of parallelograms.
- Recognize and apply properties of the diagonals of parallelograms.

THEOREM 6.8

Each diagonal of a parallelogram separates the parallelogram into two congruent triangles.

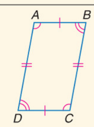
Abbreviation: *Diag. separates ▱ into 2 ≅ △s.*

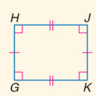
Example: $\triangle ACD \cong \triangle CAB$



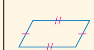
THEOREMS

		Examples
6.3	Opposite sides of a parallelogram are congruent. **Abbreviation:** *Opp. sides of ▱ are ≅.*	$\overline{AB} \cong \overline{DC}$, $\overline{AD} \cong \overline{BC}$
6.4	Opposite angles in a parallelogram are congruent. **Abbreviation:** *Opp ∠s of ▱ are ≅.*	$\angle A \cong \angle C$, $\angle B \cong \angle D$
6.5	Consecutive angles in a parallelogram are supplementary. **Abbreviation:** *Cons. ∠s in ▱ are suppl.*	$m\angle A + m\angle B = 180$, $m\angle B + m\angle C = 180$, $m\angle C + m\angle D = 180$, $m\angle D + m\angle A = 180$
6.6	If a parallelogram has one right angle, it has four right angles. **Abbreviation:** *If ▱ has 1 rt. ∠, it has 4 rt. ∠s.*	$m\angle G = 90$, $m\angle H = 90$, $m\angle J = 90$, $m\angle K = 90$





THEOREMS — *Proving Parallelograms*

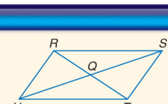
		Example
6.9	If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram. **Abbreviation:** *If both pairs of opp. sides are ≅, then quad. is ▱.*	
6.10	If both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram. **Abbreviation:** *If both pairs of opp. ∠s are ≅, then quad. is ▱.*	
6.11	If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram. **Abbreviation:** *If diag. bisect each other, then quad. is ▱.*	
6.12	If one pair of opposite sides of a quadrilateral is both parallel and congruent, then the quadrilateral is a parallelogram. **Abbreviation:** *If one pair of opp. sides is ∥ and ≅, then the quad. is a ▱.*	

THEOREM 6.7

The diagonals of a parallelogram bisect each other.

Abbreviation: *Diag. of ▱ bisect each other.*

Example: $\overline{RQ} \cong \overline{QT}$ and $\overline{SQ} \cong \overline{QU}$



CONCEPT SUMMARY — *Tests for a Parallelogram*

1. Both pairs of opposite sides are parallel. (Definition)
2. Both pairs of opposite sides are congruent. (Theorem 6.9)
3. Both pairs of opposite angles are congruent. (Theorem 6.10)
4. Diagonals bisect each other. (Theorem 6.11)
5. A pair of opposite sides is both parallel and congruent. (Theorem 6.12)

6-4 Rectangles

THEOREM 6.13

If a parallelogram is a rectangle, then the diagonals are congruent.

Abbreviation: *If ▭ is rectangle, diag. are ≅.*

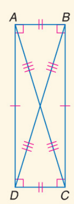
$\overline{AC} \cong \overline{BD}$

6-4 Rectangles

KEY CONCEPT *Rectangle*

Words A rectangle is a quadrilateral with four right angles.

Properties	Examples
1. Opposite sides are congruent and parallel.	$\overline{AB} \cong \overline{DC}$ $\overline{AB} \parallel \overline{DC}$ $\overline{BC} \cong \overline{AD}$ $\overline{BC} \parallel \overline{AD}$
2. Opposite angles are congruent.	$\angle A \cong \angle C$ $\angle B \cong \angle D$
3. Consecutive angles are supplementary.	$m\angle A + m\angle B = 180$ $m\angle B + m\angle C = 180$ $m\angle C + m\angle D = 180$ $m\angle D + m\angle A = 180$
4. Diagonals are congruent and bisect each other.	$\overline{AC} \cong \overline{BD}$ $\overline{AC}$ and $\overline{BD}$ bisect each other.
5. All four angles are right angles.	$m\angle DAB = m\angle BCD =$ $m\angle ABC = m\angle ADC = 90$

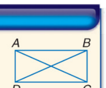


6-4 Rectangles

THEOREM 6.14

If the diagonals of a parallelogram are congruent, then the parallelogram is a rectangle.

Abbreviation: *If diagonals of ▭ are ≅, ▭ is a rectangle.*

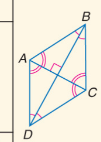


$\overline{AC} \cong \overline{BD}$

6-5 Rhombi and Squares

THEOREMS *Rhombus*

		Examples
6.15	The diagonals of a rhombus are perpendicular.	$\overline{AC} \perp \overline{BD}$
6.16	If the diagonals of a parallelogram are perpendicular, then the parallelogram is a rhombus. (Converse of Theorem 6.15)	If $\overline{BD} \perp \overline{AC}$ then ▱ABCD is a rhombus.
6.17	Each diagonal of a rhombus bisects a pair of opposite angles.	$\angle DAC \cong \angle BAC \cong \angle DCA \cong \angle BCA$ $\angle ABD \cong \angle CBD \cong \angle ADB \cong \angle CDB$



6-5 Rhombi and Squares

CONCEPT SUMMARY *Properties of Rhombi and Squares*

Rhombi

1. A rhombus has all the properties of a parallelogram.
2. All sides are congruent.
3. Diagonals are perpendicular.
4. Diagonals bisect the angles of the rhombus.

Squares

1. A square has all the properties of a parallelogram.
2. A square has all the properties of a rectangle.
3. A square has all the properties of a rhombus.

6-6 Trapezoids

THEOREMS *Isosceles Trapezoid*

6.18 Each pair of base angles of an isosceles trapezoid are congruent.

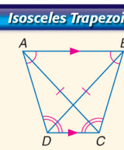
6.19 The diagonals of an isosceles trapezoid are congruent.

Example:

$\angle DAB \cong \angle CBA$

$\angle ADC \cong \angle BCD$

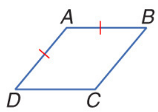
$\overline{AC} \cong \overline{BD}$



6-2 Parallelograms

EXAMPLE Proof of Theorem 6.4

1 **Prove that if a parallelogram has two consecutive sides congruent, it has four sides congruent.**



Given: $\square ABCD$; $\overline{AD} \cong \overline{AB}$

Prove: $\overline{AD} \cong \overline{AB} \cong \overline{BC} \cong \overline{CD}$

6-2 Parallelograms

CHECK Your Progress

1 **Proof:**

Statements	Reasons
1. $\square ABCD$	1. Given
2. $\overline{BC} \cong \overline{DA}, \overline{AB} \cong \overline{CD}$	2. Opposite sides of a parallelogram are congruent.
3. $\angle ABD \cong \angle CDB, \angle BAC \cong \angle DCA$ $\angle CBD \cong \angle ADB, \angle BCA \cong \angle DAC$	3. If 2 ∥ lines are cut by a transversal, alternate interior ∠s are ≅.
4. $\Delta BEC \cong \Delta DEA$, $\Delta BEA \cong \Delta DEC$	4. ______?________

A. SSS B. ASA

C. SAS D. AAS

CheckPoint

6-2 Parallelograms

EXAMPLE Proof of Theorem 6.4

1 **Proof:**

Statements	Reasons
1. $\square ABCD$	1. Given
2. $\overline{AD} \cong \overline{AB}$	2. Given
3. $\overline{CD} \cong \overline{AB}, \overline{BC} \cong \overline{AD}$	3. Opposite sides of a parallelogram are ≅.
4. $\overline{AD} \cong \overline{AB} \cong \overline{BC} \cong \overline{CD}$	4. Transitive Property

---

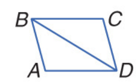
6-3 Tests for Parallelograms

EXAMPLE Write a Proof

1 **Write a 2 column proof:**

Given: $\Delta ABD \cong \Delta CDB$

Prove: *ABCD* is a parallelogram.



Statements	Reasons
1. $\Delta ABD \cong \Delta CDB$	1. Given
2. $AB \cong CD$	2. CPCTC
3. $AD \cong CB$	3. CPCTC
4. ∴*ABCD* is a parallelogram.	4. If both pairs of opposite sides of a quad are congruent, then the quad is parallelogram.

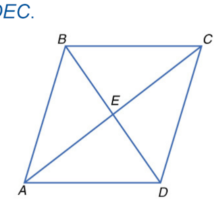
6-2 Parallelograms

CHECK Your Progress

1 **Prove that if** AC **and** BD **are the diagonals of** $\square ABCD$, $\triangle BEC \cong \triangle DEA$ **and** $\triangle BEA \cong \triangle DEC$.

Given: $\square ABCD$

Prove: $\triangle BEC \cong \triangle DEA$
$\triangle BEA \cong \triangle DEC$

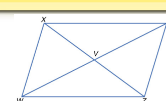


Choose which reason best completes the following proof.

CheckPoint

6-3 Tests for Parallelograms

CHECK Your Progress



1 **Write a 2 column proof:**

Given: $\Delta XVY \cong \Delta ZVW$ and $\Delta XVW \cong \Delta ZVY$

Prove: *WXYZ* is a parallelogram.

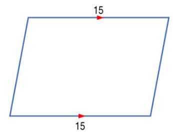
Statements	Reasons
1. ΔXVY ≅ ΔZVW	1. Given
2. ΔXVW ≅ ΔZVY	2. Given
3. XV ≅ ZV	3. CPCTC
4. VW ≅ VY	4. CPCTC
5. V is the midpoint of XZ and WY.	5. Definition of a Midpoint
6. Diagonals XZ and WY bisect each other.	6. Definition of Bisector
7. ∴WXYZ is a parallelogram.	7. If the diagonals of quadrilateral bisect each other, then the quadrilateral is a parallelogram.

CheckPoint

6-3 Tests for Parallelograms

CHECK Your Progress

3 Which method would prove the quadrilateral is a parallelogram?



- **A.** **Both pairs of opp. sides ||.**
- **B.** **Both pairs of opp. sides ≅.**
- **C.** **Both pairs of opp. ∠'s ≅.**
- **D.** **One pair of opp. sides both || and ≅.**

CheckPoint

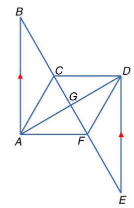
6-5 Rhombi and Squares

CHECK Your Progress

1 Complete the following proof.

Given: *ACDF* **is a rhombus;** $\overline{AB} \parallel \overline{DE}$.

Prove: $\Delta ABG \cong \Delta DEG$

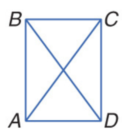


CheckPoint

6-4 Rectangles

Real-World EXAMPLE **Diagonals of a Parallelogram**

3 Kyle is building a barn for his horse. He measures the diagonals of the door opening to make sure that they bisect each other and they are congruent. How does he know that the measure of each corner is 90°?



Answer: We know that $\overline{AC} \cong \overline{BD}$. A parallelogram with congruent diagonals is a rectangle. Therefore, the corners are 90° angles.

6-5 Rhombi and Squares

CHECK Your Progress

1

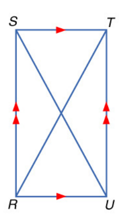
Statements	Reasons
1. *ACDF* is a rhombus and $\overline{AB} \parallel \overline{DE}$	1. Given
2. $\angle AGB \cong \angle DGE$	2. Vertical Angles Theorem
3. $\overline{AG} \cong \overline{DG}$	3. Diagonals of a rhombus bisect each other.
4. $\angle BAG \cong \angle EDG$	4. Alternate Interior Angles Theorem
5. $\Delta ABG \cong \Delta DEG$	5. ____?____

CheckPoint

6-4 Rectangles

CHECK Your Progress

3 Max is building a swimming pool in his backyard. He measures the length and width of the pool so that opposite sides are parallel. He also measures the diagonals of the pool to make sure that they are congruent. How does he know that the measure of each corner is 90?



- **A.** Since opp. sides are ||, *STUR* must be a rectangle.
- **B.** Since opp. sides are ≅, *STUR* must be a rectangle.
- **C.** Since diagonals of the ▱ are ≅, *STUR* must be a rectangle.
- **D.** *STUR* is not a rectangle.

CheckPoint

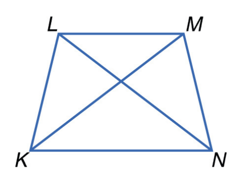
6-6 Trapezoids

EXAMPLE **Proof of Theorem 6.19**

1 Write a flow proof.

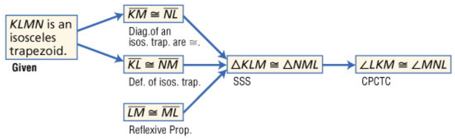
Given: *KLMN* is an isosceles trapezoid.

Prove: $\angle LKM \cong \angle MNL$



EXAMPLE Proof of Theorem 6.19

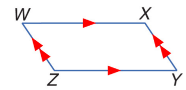
1 **Proof:**



Five-Minute CHECK (over Lesson 6-2)

Standardized Test Practice

6 **Refer to the figure. Which congruence statement is not necessarily true if *WXYZ* is a parallelogram?**



A. $\overline{WY} \cong \overline{XZ}$

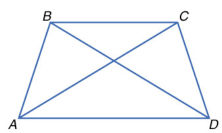
B. $\overline{WX} \cong \overline{YZ}$

C. $\angle W \cong \angle Y$

D. $\angle X \cong \angle Z$

CHECK Your Progress

1 **Write a flow proof.**

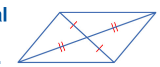


Given: *ABCD* is an isosceles trapezoid.

Prove: $\angle CBD \cong \angle BCA$

Five-Minute CHECK (over Lesson 6-3)

1 **Determine whether the quadrilateral shown in the figure is a parallelogram. Justify your answer.**



A. Yes; diagonals bisect each other.

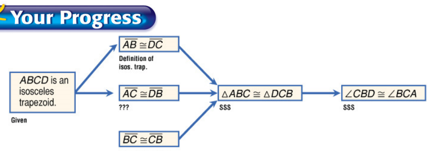
B. Yes; both pairs of opposite angles are congruent.

C. No; opposite sides are not congruent.

D. No; diagonals are not congruent.

CHECK Your Progress

1 **Proof:**



Which reason best completes the flow proof?

A. Substitution

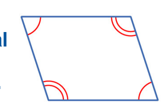
B. Definition of trapezoid

C. CPCTC

D. Diagonals of an isosceles trapezoid are ≅.

Five-Minute CHECK (over Lesson 6-3)

2 **Determine whether the quadrilateral shown in the figure is a parallelogram. Justify your answer.**



A. Yes; diagonals bisect each other.

B. Yes; both pairs of opposite angles are congruent.

C. No; opposite sides are not congruent.

D. No; diagonals are not congruent.

Quadrilaterals

CHAPTER 6

Five-Minute CHECK (over Lesson 6-3)

Standardized Test Practice

5 Which set of statements will prove that *LMNO* shown in the figure is a parallelogram?

A. $\overline{LM} \parallel \overline{NO}$ and $\overline{LM} \cong \overline{MN}$

B. $\overline{LO} \parallel \overline{MN}$ and $\overline{LO} \cong \overline{MN}$

C. $\overline{LM} \cong \overline{LO}$ and $\overline{ON} \cong \overline{MN}$

D. $\overline{LO} \cong \overline{MN}$ and $\overline{LO} \cong \overline{ON}$

0%

A B C D

CheckPoint

Quadrilaterals

CHAPTER 6

Five-Minute CHECK (over Lesson 6-5)

Standardized Test Practice

6 What property applies to a square, but not a rhombus?

A. Opposite angles are congruent.

B. Opposite sides are congruent.

C. Diagonals bisect each other.

D. All angles are right angles.

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A B C D

CheckPoint